SMART SOCIETY INTERNATIONAL MATHEMATICS OLYMPIAD (SSIMO)

Organized by

Department of Basic Science and Humanities
INSTITUTE OF ENGINEERING & MANAGEMENT,
IEM-UEM GROUP and SMART SOCIETY, USA

XI CLASS

TOTAL QUESTIONS: 50 **DURATION: 1 HOUR**

INSTRUCTIONS TO THE CANDIDATES:

1. The following question paper is divided into 3 sections
   a) Logical Reasoning (15 Questions)
   b) Mathematics (10 Questions)
   c) Achiever's Level Mathematics (25 Questions)
2. Each Logical Reasoning question carries 1 mark. Each Mathematics question carries 2 marks and each Achiever's Level Mathematics carries 3 marks.
3. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
4. There is only one correct answer. Choose only ONE option for an answer.
5. To mark your choice of answers by darkening the circles on the OMR sheet, Use HB Pencil or Blue/Black ball point pen.
6. Rough work should be done in the blank space provided in booklet.
7. Return the OMR sheet to the invigilator at the end of exam.
8. Please fill in your personal details in the space provided below.

NAME: .....

SCHOOL NAME: .....

CONTACT DETAILS OF GUARDIAN: .....

SECTION A LOGICAL REASONING

1. A man walks 1 km towards East and then he turns to South and walks 5 km. Again he turns to East and walks 2 km, after this he turns to North and walks 9 km. Now, how far is he from his starting point?

   a) 3 km b) 4 km

   c) 5 km d) 8 km

2. A child is looking for his father. He went 90 meters in the East before turning to his right. He went 20 meters before turning to his right again to look for his father at his uncle's place 30 meters from this point. His father was not there. From here he went 100 meters to the North before meeting his father in a street How far did the son meet his father from the starting point?

   a) 80 meters b) 100 meters

   c) 140 meters d) 260 meters

3. Number of letters skipped in between adjacent letters in the series is two. Which of the following series observes this rule?

   a) MPSVYBE b) QSVYZCF

   c) SVZCGJN d) ZCGKMPR

4. Number of letters skipped in between adjacent letters in the series is in the order of l2, 22, 32. Which of the following series observes the rule given above?

   a) CEJT b) EGLO

   c) EG LP d) RTWZ

5. Select that series in which letters are not according to a general rule.

   a) CEGIKM b) MORTVX

   c) PRTVXZ d) ZBDFHJ

6. If it is possible to make a meaningful word with the first, the third, the seventh and the ninth letters of the word SEPARATION, which of the following will be the third letter of that word? If no such word can be made, give X as the answer and if more than one such word can be made, give M as the answer.

   a) O b) P

   c) X d) M

7. Nitin was counting down from 32.Sumit was counting upwards the numbers starting from 1 and he was calling out only the odd numbers. What common number will they call out at the same time if they were calling out at the same speed?

   a) 19

   b) 21

   c) 22

   d) They will not call out the same number

8. How many numbers from 1 to 100 are there each of which is not only exactly divisible by 4 but also has 4 as a digit?

   a) 7 b) 10

   c) 20 d) 21

9. How many numbers amongst the numbers 9 to 54 are there which are exactly divisible by 9 but not by 3?

   a) 8 b) 6

   c) 5 d) Nil

10. The train for Lucknow leaves every two and a half hours from New Delhi Railway Station. An announcement was made at the station that the train for Lucknow had left 40 minutes ago and the next train will leave at 18.00 hrs. At what time was the announcement made?

    a) 15.30 hrs b) 17.10 hrs

    c) 16.00 hrs d) None of these

11. If 18th February, 1997 falls on Tuesday, then what will be the day on 18th February, 1999?

a) Monday b) Tuesday

c) Thursday d) Friday

12. The number of boys in a class is three times the number of girls. Which one of the following numbers cannot represent the total number of children in the class?

a) 48 b) 44

c) 42 d) 40

13. In a class, 20% of the members own only two cars each, 40% of the remaining own three cars each and the remaining members own only one car each. Which of the following statements is definitely true from the given statements?

a) Only 20% of the total members own three cars each.

b) 48% of the total members own only one car each.

c) 60% of the total members own at least two cars each.

d) 80% of the total members own at least one car.

14. Robin says, "If Jai gives me Rs 40, he will have half as much as Atul, but if Atul gives me Rs 40, then the three of us will all have the same amount." What is the total amount of money that Robin, Jai and Atul have between them?

a) Rs 240 b) Rs 320

c) Rs 360 d) Rs 420

15. A, B, C, D and E play a game of cards. A says to B, "If you give me three cards! you will have as many as E has and if I give you three cards, you will have as many as D has." A and B together have 10 cards more than what D and E together have. If B has two cards more than what C has and the total number of cards be 133, how many cards does B have?

a) 22 b) 23

c) 25 d) 35

SECTION B

MATHEMATICAL REASONING

16. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

a) 601 b) 600

c) 603 d) 602

17. A circle touches the x-axis and also touches the circle with center at (0, 3) and radius 2. The locus of the center of the circle is

a) Ellipse b) Circle

c) Hyperbola d) Parabola

18. An ellipse has OB as semi minor axis, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$

c) $\frac{1}{4}$ d) $\frac{1}{\sqrt{3}}$

19. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2(x)+5\sin(x)-3=0$ is

a) 4 b) 6

c) 1 d) 2

20. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

a) 5040 b) 6210

c) 385 d) 1110

21. Let C be the circle with center (0,0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre

a) $x^2+y^2=\frac{3}{2}$ b) $x^2+y^2=1$

c) $x^2+y^2=\frac{9}{4}$ d) $x^2+y^2=\frac{3}{5}$

22. The number of real solutions of $1+|e^x-1|=e^x(e^x-2)$ is:

a) 0 b) 1

c) 3 d) 2

23. The complex number z satisfying the equations $|\frac{z-12}{z-8i}|=\frac{5}{3}$ and $|\frac{z-4}{z-8}|=1$

a) 6+8i b) 6+17i

c) 5+8i d) Both a) and b)

24. The equation of the line, which passes through the point (3,-2) and are inclined at 60° to the line $\sqrt{3}x+y=1$ is

a) y+2=0 b) x+2=0

c) $\sqrt{3}x-y=2+3\sqrt{3}$ d) Both a) and c)

25. The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2=4ax$ is another parabola with the directrix

a) x=0 b) y=0

c) $x=-\frac{a}{2}$ d) $y=-\frac{a}{2}$

SECTION C

ACHIEVER'S LEVEL

26. Let α and β be the roots of $x^2-6x-2=0$, with $\alpha>\beta$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is

a) 1 b) 2

c) 3 d) 4

27. Let $a_1, a_2, a_3, \ldots$ be an A.P with $a_6=2$. Then the common difference of this A.P, which maximises the product $a_1a_4a_5$ is

a) 3/2 b) 8/5

c) 6/5 d) 2/3

28. If ab = 2a + 3b, a > 0, b > 0 then the minimum value of ab is

a) 12 b) 24

c) 1/4 d) 2/3

29. If $x^2+px-444p=0$ has integral roots, where p is a prime number, then the value(s) of p is (are)

a) 2 b) 3

c) 2, 3 and 37 d) 37

30. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller to the numbers on the removed card is k, then k-20 equals

a) 5 b) 10

c) 15 d) 50

31. If the sum of the first n terms of an AP is cn^2, then the sum of squares of these n terms is

a) $\frac{n(4n^2-1)c^2}{6}$

b) $\frac{n(4n^2+1)c^2}{3}$

c) $\frac{n(4n^2-1)c^2}{3}$

d) $\frac{n(4n^2+1)c^2}{6}$

32. The number of terms in an AP is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by then $10\frac{1}{2}$ the number of terms in the AP is:

a) 4 b) 8

c) 12 d) 16

33. For $x \in (0,\pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

a) Infinitely many solutions

b) 3 solutions

c) 1 solution

d) No solution

34. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

a) 1/3 b) 2/9

c) -7/9 d) -3/5

35. The value of $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0$ is

a) 1 b) 4

c) 2 d) 0

36. The center of the circle inscribed in the square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

a) (4, 7) b) (7, 4)

c) (9, 4) d) (4, 9)

37. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is

a) $x = -1$ b) $x = 1$

c) $x = -\frac{3}{2}$

d) $x = \frac{3}{2}$

38. The length of minor axis (along y-axis) of an ellipse of the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line $x + 6y = 8$, then its eccentricity is:

a) $\sqrt{\frac{5}{12}}$ b) $\sqrt{\frac{11}{12}}$

c) $\sqrt{\frac{5}{6}}$ d) $\sqrt{\frac{11}{27}}$

39. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions is:

a) 2250 b) 2255

c) 1500 d) 3000

40. If $\left|\frac{z+i}{z-i}\right| = \sqrt{3}$, then radius of the circle is

a) $\frac{2}{\sqrt{21}}$ b) $\frac{1}{\sqrt{21}}$

c) $\sqrt{3}$ d) $\sqrt{21}$

41. If α and $\beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

1) 1 2) 2

3) -1 4) 0

42. $\lim_{x\to 2} \frac{3^x + 3^{3-x} - 12}{3^{-\frac{x}{2}} - 3^{1-x}}$ is equal to

a) 36 b) 27

c) 1 d) -1

43. If a function f is such that $f(0)=2$, $f(1)=3$, $f(x+2)=2f(x)-f(x+1)$ for $x \geq 0$, then $f(5)$ is equal to

a) -7 b) -3

c) 17 d) 13

44. If $A+B+C=180^0$ then the value of $\tan A+\tan B+\tan C$ is

a) $\geq 3\sqrt{3}$ b) $\geq 2\sqrt{3}$

c) $> 3\sqrt{3}$ d) $> 2\sqrt{3}$

45. If $x=\frac{1-\sqrt{y}}{1+\sqrt{y}}$ then $\frac{dy}{dx}$ is equal to

a) $\frac{4}{(x+1)^2}$ b) $\frac{4(x-1)}{(x+1)^3}$

c) $\frac{(x-1)}{(x+1)^3}$ d) $\frac{4}{(x+1)^3}$

46. For three events A, B, and C, P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of C or A occurs) = $\frac{1}{4}$ and P (All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs is:

a) $\frac{7}{16}$ b) $\frac{7}{64}$

c) $\frac{3}{16}$ d) $\frac{7}{32}$

47. A debate club consists of 6 girls and 4 boys. A team of 4 members is said to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to be included at most one boy, then the number of ways of selecting the team is

a) 380 b) 320

c) 260 d) 95

48. If the tangent at the point P on the circle is $x^2+y^2+6x+6y=2$ meets a straight line $5x-2y+6=0$ at a point Q on the y-axis, then the length of PQ is

a) 4 b) 5

c) $2\sqrt{5}$ d) $3\sqrt{5}$

49. If $2x-y+1=0$ is a tangent to the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{16}=1$$

then which of the following can be sides of a right angled triangle?

a) a, 4, 1 b) a, 4, 2

c) 2a, 8, 1 d) 2a, 4, 1

50. If a triangle ABC has vertices A(-1, 7), B(-7, 1), and C(5, -5), then its orthocenter has coordinates

a) (-3,3)

b) $\left(-\frac{3}{5},\frac{3}{5}\right)$

c) $\left(\frac{3}{5},-\frac{3}{5}\right)$

d) (3,-3)