

# SMART SOCIETY INTERNATIONAL MATHEMATICS OLYMPIAD (SSIMO)



## Organized by

Department of Basic Science and Humanities  
INSTITUTE OF ENGINEERING & MANAGEMENT,  
IEM-UEM GROUP and SMART SOCIETY, USA

**TOTAL QUESTIONS: 50**

**DURATION: 1 HOUR**

## INSTRUCTIONS TO THE CANDIDATES:

1. The following question paper is divided into 3 sections
  - a) Logical Reasoning (15 Questions)
  - b) Mathematics (10 Questions)
  - c) Achiever's Level Mathematics (25 Questions)
2. Each Logical Reasoning question carries 1 mark. Each Mathematics question carries 2 marks and each Achiever's Level Mathematics carries 3 marks.
3. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
4. There is only one correct answer. Choose only ONE option for an answer.
5. To mark your choice of answers by darkening the circles on the OMR sheet, Use HB Pencil or Blue/Black ball point pen.
6. Rough work should be done in the blank space provided in booklet.
7. Return the OMR sheet to the invigilator at the end of exam.
8. Please fill in your personal details in the space provided below.

NAME: .....

SCHOOL NAME: .....

CONTACT DETAILS OF GUARDIAN: .....

## SECTION A LOGICAL REASONING

- In a certain code BRAVE is coded as 20, COMMON is coded as 30. How is KISHORE written in that code?  
a) 85                      b) 42  
c) 75                      d) 40
- A truck travels 36km North, then it turns West and travels 9km, then it turns south and travels 50km, then it turns to its left and travel 9km. where is it now with reference to its starting point?  
a) 14km North      b) 86km South  
c) 86km North      d) 14km South
- A 5-digit number  $xy235$  is divisible by 3 such that  $x+y < 5$ , where  $x$  and  $y$  are digits, then possible values of  $(x,y)$  are  
a) (1, 1) or (4, 0)    b) (1, 1) or (2, 0)  
c) (1, 1) or (0, 2)    d) (2, 0) or (0, 2)
- Find the number  
12, 6, 15, 67.5, \_\_\_\_\_, 3729.375  
a) 583.75              b) 589.75  
c) 498.75              d) 438.75
- In a question on division with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is  
a) 0                      b) 12  
c) 13                      d) 20
- In a certain code, 256 means you are good, 637 means we are bad and 358 means good and bad. Which of the following does represent and in that c  
a) 2                      b) 5  
c) 8                      d) 3
- In a certain code language, (i) 'il be pee' means 'rases are blue'; (ii) 'sik nee' means 'red flowers'; (iii) 'pee mithee' means 'flowers are vegetables'. How is 'vegetables are red flowers' written in that code?  
a) pee sikmithee  
b) sik pee hee be  
c) il sikmithee  
d) cannot be determined
- Four girls are sitting on a bench to be photographed. Shikha is to the left of reena. Manju is to the right of Reena. Rita is between Reena and Manju. Who would be the second from the left in the photograph?  
a) Reena                  b) Shikha  
c) Manju                  d) Rita
- In a chess tournament each of the seven players will play every other player exactly once. How many matches will be played during the tournament?  
a) 12                      b) 21  
c) 14                      d) 13
- A box contains 5 red balls and 7 green balls. If two balls are drawn at random, what is the probability that both balls are green?  
a)  $\frac{7}{66}$                       b)  $\frac{21}{66}$   
c)  $\frac{1}{11}$                       d)  $\frac{1}{6}$
- A father is 4 times as old as his son. After 20 years, he will be twice as old as his son. What is the present age of the son?  
a) 10                      b) 20  
c) 15                      d) 25

12. If it was Saturday on 17th November, 1962, what will be the day on 22nd November, 1964?

- a) Monday      b) Tuesday  
c) Wednesday      d) Sunday

13. If '+' means 'x', 'x' means '-', '-' means '+' and '-' means '+', then  $175 - 25 \div 5 + 20 \times 3 + 10 = ?$

- a) 16      b) 77  
c) 2370      d) 240

14. Ajay left home for the bus stop 15 minutes earlier than usual. It takes 10 minutes to reach the stop. He reached the stop at 8.40 a.m. What time does he usually leave home for the bus stop?

- a) 8.30 a.m.      b) 8.45 a.m.  
c) 8.55 a.m.      d) Data inadequate

15. What is the value of A if each letter represents a different digit?

$$\begin{array}{r} A \ 3 \ B \\ \times B \\ \hline 2 \ 1 \ 7 \ B \end{array}$$

- a) 3      b) 4  
c) 5      d) 7

## SECTION B

### MATHEMATICAL REASONING

16. If  $P(E) = 0.05$ , what is the probability of 'not E'?

- a) 0.97      b) 0.95  
c) 0.85      d) 0.87

17. If it is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that 2 students have the same birthday?

- a) 0.006      b) 0.08  
c) 0.06      d) 0.008

18. Find the quadratic polynomial with  $0, \sqrt{5}$  as the sum and product of its zeroes respectively.

- a)  $x^2 + \sqrt{3}$       b)  $x^2 + \sqrt{9}$   
c)  $x^2 + \sqrt{7}$       d)  $x^2 + \sqrt{5}$

19. Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73.

- a) 187      b) 178  
c) 158      d) 167

20. If  $\sec \theta = \frac{13}{5}$ , then  $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} =$

- a) 5      b) 3  
c) 6      d) 4

21. Find the volume of the largest right circular cone that can be cut out of a cube where edge is 9cm?

- a)  $180.93\text{cm}^3$       b)  $185\text{cm}^3$   
c)  $190.93\text{cm}^3$       d)  $195\text{cm}^3$

22. A and B each have a certain number of coins. A says to B, 'if you give me 20 of your coins, I will have twice as many as left with you'. B replies, if

you give me 25 of your coins I will have thrice as many as left with you'. The number of coins do A and B have respectively

- a) 52, 56                      b) 42, 66  
c) 52, 46                      d) 42, 56

23. At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth grades and twice as many fourth graders as fifth graders. What the average number of minutes run per day by these students?

- a) 12                              b)  $37/3$   
c)  $88/7$                           d) 14

24. Externally tangent circles with centers at points A and B have radii of lengths 5 and 3 respectively. A line externally tangent both circles intersect ray AB at point C. What is BC?

- a) 4                                b) 4.8  
c) 10.2                          d) 12

25. A pair of six-sided dice are labeled so that die has only even numbers (two each of 2, 4 and 6), and the other die has only odd number (two of each 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7?

- a)  $1/6$ .                            b)  $1/5$   
c)  $1/2$                             d)  $1/3$

## SECTION C

### ACHIEVER'S LEVEL

26. Three cubes of a metal whose edges are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is  $12\sqrt{3}$ . Find the edges of three cubes.

- a) 6, 8, 10 cm                  b) 7.5, 10, 12.5 cm  
c) 9, 12, 15 cm                d) 3, 4, 5 cm

27. The rain water from a  $22\text{m} \times 20\text{m}$  roof drains into a cylindrical vessel of diameter 2m and height 3.5m. If the rain water collected from the roof fills  $4/5$  th of the cylindrical vessel, then find the rainfall in centimeter.

- a) 4 cm                            b) 2 cm  
c) 6 cm                            d) 5 cm

28. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4cm, 5cm, and 7cm respectively. Determine the length of fourth side.

- a) 4 cm                            b) 2 cm  
c) 6 cm                            d) 5 cm

29. If the tangent at point P to the circle with center O cuts a line through O and Q such that  $PQ=24$  cm and  $OQ=25$  cm. Find the radius of circle.

- a) 6 cm                            b) 2 cm  
c) 7 cm                            d) 10 cm

30. A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of length 14cm and 16cm respectively. If area of  $\Delta PQR$  is 336 cm, find the side PQ and PR.

- a) 24, 26 cm                      b) 26, 24 cm  
c) 28, 26 cm                      d) 26, 28 cm

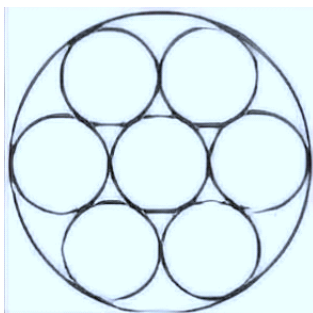
31. From a window of a house, which is  $x$  metres high from the ground, the angles of elevation and depression of the top and foot of another building on the opposite side of the road are  $\alpha$  and  $\beta$  respectively. Then the height of opposite building will be

- a)  $x(1 + \tan \alpha \cot \beta)$
- b)  $x(1 + \tan \alpha \tan \beta)$
- c)  $x \tan \alpha (\tan \alpha + \cot \beta)$
- d)  $x(1 + \cot \alpha \cot \beta)$

32. Find the distance between the following pair of points  $(a \sin a, -b \cos a)$  and  $(-a \cos a, b \sin a)$

- a)  $(\sin a + \cos a) \sqrt{a^2 + b^2}$
- b)  $(\sin a - \cos b) \sqrt{a^2 + b^2}$
- c)  $(\sin a - \cos a) \sqrt{a^2 + b^2}$
- d)  $(\sin b + \cos a) \sqrt{a^2 + b^2}$

33. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown in the figure. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



- a)  $\sqrt{2}$
- b) 1.5
- c)  $\sqrt{\pi}$
- d)  $\sqrt{2\pi}$

34. In the given figure,  $O$  is the center and  $SAT$  is a tangent to the circle at  $A$ . If  $\angle BAT = 30^\circ$ , then find  $\angle AOB$  and  $\angle AQB$

- a)  $60^\circ, 150^\circ$
- b)  $30^\circ, 150^\circ$
- c)  $60^\circ, 60^\circ$
- d) None of these

35. The ratio of the roots of the equation  $ax^2 + bx + c = 0$  is same as the ratio of the roots of the equation  $px^2 + qx + r = 0$ . If  $D_1$  and  $D_2$  are the discriminants of  $ax^2 + bx + c = 0$  and  $px^2 + qx + r = 0$  respectively, then  $D_1 : D_2$  is

- a)  $a^2 : p^2$
- b)  $b^2 : q^2$
- c)  $c^2 : r^2$
- d) None of the above

36. Find the sum of the first 10 terms of the series 2, 5, 10, 17, .....

- a) 420
- b) 410
- c) 395
- d) 430

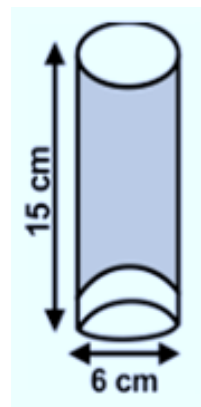
$$37. \left( \frac{\sqrt{3} + 2 \cos A}{1 - 2 \sin A} \right)^{-3} + \left( \frac{1 + 2 \sin A}{\sqrt{3} - 2 \cos A} \right)^{-3} = \text{---}$$

- a) 1
- b)  $\sqrt{3}$
- c) 0
- d) -1

38. A toothed wheel of diameter 60 cm. is attached to a smaller wheel of diameter 48 cm. How many revolutions will the smaller wheel make when the larger one makes 24 revolutions?

- a) 40
- b) 30
- c) 20
- d) 25

39. In the adjoining figure, the bottom of the glass has a hemispherical raised portion. If the glass is filled with orange juice, then find the quantity of juice which a person will get:



- a)  $135\pi$
- b)  $117\pi$
- c)  $99\pi$
- d)  $36\pi$

40. If the mean of  $k$  and  $\frac{1}{k}$  is  $A$ , then the mean of  $k^2$  and  $\frac{1}{k^2}$  will be

- a)  $2A^2 - 1$       b)  $2A^2 + 1$   
c)  $\frac{A^2}{2} - 1$       d)  $\frac{A^2}{2} + 1$

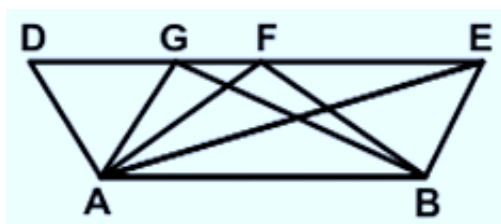
41. Two regular polygons are such that the ratio between their number of sides is 1:2 and the ratio of measures of their interior angles is 3:4. Find the number of sides of each polygon.

- a) 5, 10      b) 6, 12  
c) 4, 8      d) 8, 16

42. 4. Perpendiculars are drawn from the vertex of the obtuse angles of a rhombus to its sides. The length of each perpendicular is equal to a unit. The distance between their feet is equal to  $b$  units. Find the area of the rhombus.

- a)  $\frac{\sqrt{a^2 + b^2}}{2\sqrt{(b^2 - a^2)}}$   
b)  $\frac{ab^2}{2\sqrt{b^2 - a^2}}$   
c)  $\frac{2ab}{2\sqrt{b^2 - a^2}}$   
d)  $\frac{2a^2b^2}{2\sqrt{b^2 - a^2}}$

43. In the given figure,  $AB \parallel DE$  and the area of the parallelogram  $ABFD$  is  $24 \text{ cm}^2$ . Find the area of triangle  $AEB$ .



- a)  $8 \text{ cm}^2$       b)  $12 \text{ cm}^2$   
c)  $10 \text{ cm}^2$       d)  $14 \text{ cm}^2$

44. Consider the following frequency distribution of the heights of 60 students of a class:

Height (in cm)	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Number of students	15	13	10	8	9	5

The sum of the lower limit of the modal class and upper limit of the median class is

- a) 310      b) 315  
c) 320      d) 330

45. If  $x^5 + 2x^4 + x + 6$  is divided by  $g(x)$  and quotient is  $x^2 + 5x + 7$ , then the possible degree of remainder is:

- a) less than 1      b) less than 2  
c) less than 3      d) less than 4

46. It is given that  $\triangle ABC \sim \triangle DFE$ ,  $\angle A = 30^\circ$ ,  $\angle C = 50^\circ$ ,  $AB = 5 \text{ cm}$ ,  $AC = 8 \text{ cm}$  and  $DF = 7.5 \text{ cm}$ . Then, the following is true:

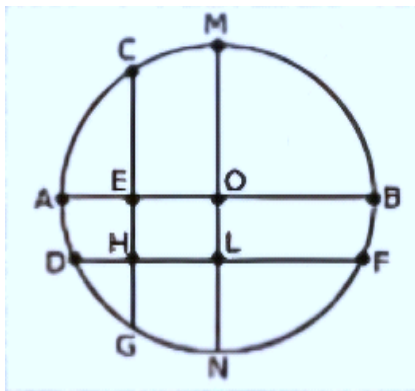
- a)  $DE = 12 \text{ cm}$ ,  $\angle F = 50^\circ$   
b)  $DE = 12 \text{ cm}$ ,  $\angle F = 100^\circ$   
c)  $EF = 12 \text{ cm}$ ,  $\angle D = 100^\circ$   
d)  $EF = 12 \text{ cm}$ ,  $\angle D = 30^\circ$

47. The integers 1, 2, ..., 40 are written on a blackboard. The following operation is then repeated 39 times: In each repetition, any two numbers, say  $a$  and  $b$ , currently on the blackboard are erased and a new number  $a+b-1$  is written. What will be the number left on the board at the end?

- a) 820      b) 821  
c) 781      d) 819

48. In the following figure, the diameter of the circle is 3 cm.  $AB$  and  $MN$  are two diameters such that  $MN$  is perpendicular to  $AB$ . In addition,  $CG$  is

perpendicular to AB such that  $AE:EB = 1:2$ , and DF is perpendicular to MN such that  $NL:LM = 1:2$ . The length of DH in cm is



- a)  $2\sqrt{2} - 1$       b)  $\frac{(2\sqrt{2}-1)}{2}$   
c)  $\frac{(3\sqrt{2}-1)}{2}$       d)  $\frac{(2\sqrt{2}-1)}{3}$

49. The numbers of common terms in the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, \_\_\_\_\_, 466 is

- a) 78                      b) 19  
c) 20                      d) 77

50. Two numbers  $x$  and  $y$  are selected successively from the numbers 1 to 10 without replacement. Then the probability that  $\frac{x}{y}$  is an integer is

- a)  $\frac{35}{42}$                       b)  $\frac{17}{90}$   
c)  $\frac{19}{90}$                       d)  $\frac{14}{143}$

